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ON THE DISTRIBUTIONAL-BASED MODELING AND ASSESSMENT OF RADIO SYSTEM NETWORKS RELIABILITY: TWO OR THREE PARAMETER WEIBULL DISTRIBUTION MODEL?

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ABSTRACT

In recent years, the utilization of reliability tests and results in decision-making processes in radio system networks have grown considerably. One of the most largely used statistical tools for lifetime distributional analysis and reliability estimation of engineering system is the Weibull distribution model. This is due to its exceptionally high tractability**.** As the Weibull distribution embroils transcendent equations, determining its parameters' fitting is also somewhat a demanding task. This research work aims to comparatively investigate and quantify the effect of a two-parameter (2P) and a three-parameter (3P) Weibull distribution functions on radio system networks reliability performance estimation. To accomplish that, life signal data acquired over the LTE broadband network interface is explored for the quantitative analysis. For the purpose of best model selection, both distributions were subjected to three goodness-of-fit (GoF) statistical tests, which are the Anderson–Darling, Kolmogorov–Smirnov, and Chi-Square tests. Based on the three statistical tests, it was found that the 3P Weibull distribution achieved the best reliability quantification and failure estimation performance in all the study locations. The results offer a concrete groundwork and framework for conducting better research in parametric modelling, estimation and analysis in future radio networks.

KEYWORDS: Radio networks, Distributional modeling, 3P Weibull model, Failure rate, Reliability

INTRODUCTION

The distributional assessment of the spatial-temporal characteristics of propagated radio frequency signals, especially through multipath propagation environment, are crucial for efficient signal coverage reliability planning and management in telecommunication systems networks (Joseph and Konyeha, 2013; Isabona *et al*., 2013; Isabona and Srivastava, 2017).

In recent years, the application and acceptance of reliability assessment tests as a means of improving system output or operational performance in the telecommunication sectors have grown considerably globally. For example, many telecommunication companies outsource the operational performance assessment of their products and services to second party servicing companies. This is to enable them control and manage the quality and reliability operational service delivery to subscribers in the competitive telecom market (Isabona and Ojuh, 2013). Even if the system operational deliverables of the telecom company is highly automated, there are still some level of variations in terms of service delivery owing to so many controllable and non-controllable factors.

The words ‘‘quality’’ and ‘‘reliability’’ are often utilize interchangeably. However, these two terms differ in meaning: whereas quality expresses the performance of a system at a particular point in time (Isabona and Srivastava, 2017; Isabona and Ojuh, 2013), reliability is concerned with the probability that a system is capable of performing its premeditated desired functions through a specified time under certain given conditions. Also, reliability testing procedures are usually more complex compared to quality testing procedures.

Nonetheless, reliability testing and quality testing both measure the “goodness” of a system. Besides, system reliability hinge on the initial system quality. Most reliability assessment tests are carried out to answer either one or both of the following important questions:

- Does the system performance meet the terms with the reliability benchmarks set by the company?
- Is one version of the system networks reliability better than the other one?

If the system has different failure assessment components and each tested components needs to be checked after every time throughout the life cycle; therefore, the reliability assessment could be very costly. As a result, companies often make every possible efforts to reduce the cost of reliability testing. In view of that, it is better to test only a sample of the system networks.

In this work, the target is to comparatively investigate and quantify the effect of a two-parameter (2P) and a three-parameter (3P) Weibull distributions functions in estimating the reliability and failure rate of a system, using life data acquired over an operational LTE cellular broadband system networks as case study. The LTE mobile broadband networks transmits at 1857 MHz frequency.

MATERIALS AND METHOD

Study Area

Federal University Lokoja campus is used as study area in this this work. The campus housed five faculties, with a population of about 5000 students. The campus is bounded by geographical coordinates of approximately 07°79′ 05″ N, 06°73′ 39″ E and its altitude lies between 57 and 65 meters above sea level. Generally, Lokoja town wherein the university is situated, is on the western bank of the River Niger close to its confluence with River Benue and sandwiched between the River and the Mount Patti. The terrain is characterized by tropical climate with wet and dry seasons. The annual rainfall is about

1150 mm, and mean annual temperature is about 27.7°C.

Data Collection, Tools and Procedure

To accomplish the study objectives, about 2000 life signal to interference noise ratio (SINR) datasets acquired over Long Term Evolution (LTE) broadband network interface was used for the investigation. The LTE network belongs to a commercial cellular telecommunication service provider, operating all over Nigeria. By means of the walk tests techniques (Joseph and Konyeha, 2013), the SINR dataset was collected over a couple of weeks, between the months of May and June of 2019 in five different places at the university campus. The main walk test tools used include the Samsung S4 Galaxy mobile phone and HP-laptop. The mobile phone was equipped with Cellular-Z, a free application software downloaded from Google play store. It is a reliable software which provide means of obtaining dual SIM mobile phone network information such as serving cell, Wifi network, serving cell signal quality and neighbouring cells information. With the aid of the global positioning system (GPS), the software also provide means of taking current location information and map tracking indoor signal coverage in log files. Thus, with the aid of the Samsung galaxy phone, running on the Cellular-Z software mode, calls were initiated at each test point until it is established and the signal strength information sent over the air interface between the eNodeB and the mobile station were read. For every location, received signal strength and SINR were measured at different time intervals. All measurements were taken in the mobile active mode .The Pictorial view of Federal University Lokoja where the data collection took place is displayed in figure 1. A twenty (20) sample size of the collected data is displayed in table 1. Furthermore, for the purpose determining the most credible one for reliability analysis, both distributions were also subjected three goodness-of-fit (GoF) statistical tests such as the Anderson–Darling, Kolmogorov–Smirnov, and Chi-Square tests.

Figure 1: A Pictorial View of the Premises of Federal University Lokoja

Table1: A Sample of Collected SINR Data size collected at different five locations within the University

Outdoor Physics Lab	Indoor Physics Lab	Outdoor University Conference Room	Indoor University Conference Room	Outdoor Needs Assessment building
24.6	14	20.2	12.2	12.2
24.6	14	20.2	12.2	12.2
24.6	14	20.2	12.2	12.2
24.6	14	20.2	12.2	12.2
16.8	14	20.2	12.2	12.2
16.8	14	20.2	12.2	12.2
16.8	14	15.8	12.2	12.2
16.8	8.8	15.8	12.2	11
16.8	8.8	15.8	12.2	11
12.6	8.8	15.8	12.2	11
12.6	8.8	15.8	12.2	11
11.8	12.2	14.8	12.2	11
11.8	12.2	14.8	12.2	12.8
11.8	12.2	14.8	12.2	12.8
13.6	12.2	14.8	12.2	12.8
13.6	12.2	14.8	12.2	12.8
13.6	9.8	13.8	12.2	12.8
13.6	9.8	13.8	12.2	9.8
13.6	9.8	13.8	11.4	9.8
18.4	9.8	13.8	11.4	14.6

Mathematical Framework for 2P and 3P Weibull distributions

The fundamental theories of reliability engineering are expressed using probability or probabilistic parameters such as random variables, density functions, and distribution functions. There exist a number of probability distribution functions for describing and fitting a statistical distribution to life reliability data. The choice distribution function for this work is the Weibull distribution. This is due to its ability to provide reasonably accurate failure analysis, and reliability forecasts with extremely small samples (Ng *et al.*, 2012, Isabona, 2019). This distribution function is named after his discoverer, Waloddi Weibull, a Swedish physicist who in 1939 utilise it to perform the modelling of breaking strength distribution of materials (Weibull, 1939). Since that time it was initiated by the author till now, its applications in physics, statistics, mathematics and engineering increased significantly.

The application of parametric Weibull distribution for different probabilistic and statistical studies are rich in the academic literature (Ng *et al.*, 2012, Isabona, 2019; Luus, and Jammer, 2005; Singh *et al.*, 2005; Markovic *et al.*, 2009; Nagatsuka, 2008; Cousineau, 2009; Jin *et al.*, 2012; Jin *et al.*, 2013; Atta-Elmanan, and Mohammed, 2015; Dumitrascu *et al.*, 2018). Weibull distributional fitting can be explored in two ways: (i) two-parameter (2P) distributions or (ii) three-parameter (3P) distributions.

In this work, the target is to comparatively quantify the effect of 2P and 3P Weibull distribution functions in estimating the reliability and failure rate of a system, using life data acquired over an operational LTE cellular broadband system networks as case study. By definition, the 3P Weibull's reliability function (rf), failure rate function (frf), mean time before failure function (mtbf), cumulative distribution function (cdf) and probability distribution function (pdf) can be expressed as (Luus, and Jammer, 2005; Singh *et al*., 2005):

$$rf\ : R_{(x,\lambda,\gamma,c)} = \exp\left[-\left(\frac{x-\gamma}{\lambda}\right)^{c}\right], x > \gamma \geq 0 \quad (1)$$

$$frf\ : h_{(x,\lambda,\gamma,c)} = \left[\frac{c}{\lambda}\left(\frac{x-\gamma}{\lambda}\right)^{c-1}\right], x > \gamma \geq 0 \quad (2)$$

$$mtbf\ : E(x,\lambda,\gamma,c) = \gamma + \lambda\Gamma\left(1+\frac{1}{c}\right), x > \gamma \geq 0 \quad (3)$$

$$cdf\ : F_{(x,\lambda,\gamma,c)} = \left[1-\exp\left(-\left(\frac{x-\gamma}{\lambda}\right)^{c}\right)\right], x > \gamma \geq 0 \quad (4)$$

$$pdf\ : f_{(x,\lambda,\gamma,c)} = h_{(x,\lambda,\gamma,c)} \times f_{(x,\lambda,\gamma,c)}, x > \gamma \geq 0 \quad (5)$$

$$= \left[\frac{c}{\lambda}\left(\frac{x-\gamma}{\lambda}\right)^{c-1} \exp\left(-\left(\frac{x-\gamma}{\lambda}\right)^{c}\right)\right], x > \gamma \geq 0 \quad (6)$$

where λ = the scale parameter, c =shape parameter, and γ =location parameter. $\Gamma(x)$ indicates the Gamma function.

For 2P Weibull distribution, γ =0 [5], and accordingly, the expressions in equation (1) to (6) reduce to :

$$rf\ : R_{(x,\lambda,c)} = \exp\left[-\left(\frac{x}{\lambda}\right)^{c}\right], x > 0 \quad (7)$$

$$frf\ : h_{(x,\lambda,c)} = \left[\frac{c}{\lambda}\left(\frac{x}{\lambda}\right)^{c-1}\right], x > 0 \quad (8)$$

$$mtbf\ : E(x,\lambda,c) = \lambda\Gamma\left(1+\frac{1}{c}\right), x > 0 \quad (9)$$

$$cdf\ : F_{(x,\lambda,c)} = \left[1-\exp\left(-\left(\frac{x}{\lambda}\right)^{c}\right)\right], x > 0 \quad (10)$$

$$pdf\ : f_{(x,\lambda,c)} = h_{(x,\lambda,c)} \times f_{(x,\lambda,c)}, x > 0 \quad (11)$$

$$= \left[\frac{c}{\lambda} \left(\frac{x}{\lambda} \right)^{c-1} \exp\left(-\left(\frac{x}{\lambda} \right)^{c} \right) \right], x > 0 \qquad (12)$$

Parameter Estimation Method

Assuming $x_1, x_2, x_3, x_4, ...x_n$, represent the random dataset number explored in this work, the 2P Weibull distribution in equation (12) can be written in terms of the n dataset by:

$$f(x, c, \lambda) = \left[\prod_{i=1}^{n} \frac{c}{\lambda} \left(\frac{x_i}{\lambda} \right)^{c-1} \exp\left(-\left(\frac{x_i}{\lambda} \right)^{c} \right) \right], x > 0 \qquad (13)$$

To obtain scale parameter λ and the location parameter c , from the 2P Weibull distribution in equation (13), we have,

$$\frac{\partial}{\partial \lambda} \ln\left(f(x, c, \lambda) \right) = \frac{\partial}{\partial \lambda} \left[\prod_{i=1}^{n} \frac{c}{\lambda} \left(\frac{x_i}{\lambda} \right)^{c-1} \exp\left(-\left(\frac{x_i}{\lambda} \right)^{c} \right) \right] = 0 \qquad (14)$$

$$\frac{\partial}{\partial c} \ln\left(f(x, c, \lambda) \right) = \frac{\partial}{\partial c} \left[\prod_{i=1}^{n} \frac{c}{\lambda} \left(\frac{x_i}{\lambda} \right)^{c-1} \exp\left(-\left(\frac{x_i}{\lambda} \right)^{c} \right) \right] = 0 \qquad (15)$$

The simplification of the differential expressions in equation (14) and (15) result to:

$$\lambda = \frac{1}{n} \sum_{i=1}^{n} x_i^c \left(\frac{x}{\lambda} \right)^{1/c} \qquad (16)$$

$$c = \left[\frac{\sum_i x_i^c \ln x_i}{\sum x_i^c} - \frac{\sum_i x_i}{n} \right]^{-1} \qquad (17)$$

RESULTS AND DISCUSSION

This section present the results and analysis of this work. All the computed reliability and failure rate estimates in the five study locations (L1 to L5) and as well their graphics, were implemented using Matlab 2018a software, SatAssist software and Microsoft Excel software. Shown in table 2 are the life signal characteristics obtained based on the 2P and 3P Weibull distributions using parametric maximum likelihood estimation approach. The corresponding estimated reliability, failure rate and mean time before failure values obtained over the five study locations are displayed in Figures 2 to 4 and tables 2 to 7. The results in Figure 2 showed that higher reliability estimates were obtained in L1 to L5 using 3P Weibull model compared to the 2P Weibull model. For example as summarised in tables 3 to 7, while the 3P Weibull model provided up to 99.43, 70.60, 75.35, 93.49 and 73.73% network reliability performance estimates at in L1 to L5, the 2P Weibull model attained 53.40, 51.11, 54.04, 54.10 and 48.35%, reliability performance estimates at the same locations. This simply implies about 4, 11, 11, 3, and 10% failure rate estimates with 3P Weibull model compared to 2P Weibull model that attained 22, 13, 21, 21 and 10% failure rates as shown in figure 3. For mean time before failure estimates, the reverse is the case with 3P compared to 2P Weibull model. These estimated failure rate values also indicate somewhat a high level transmission quality for the LTE radio networks in locations 1 and 5 within the university campus. On the other hand, but poor transmission and reliability weaknesses are observed in locations L2, L3 and L4. Such weaknesses can be eliminated by means of radio link adaptation through transmit power control (Sattiraju, and Schotten, 2014; Isabona *et al*, 2008), load control strategies (Isabona and Ekpenyong, 2008; Igbonovia *et al*, 2017); call admission control (Ekpenyong and Isabona, 2011); multipath antenna diversity (Ekpenyong *et al.*, 2017 and dynamic antenna tilting (Dandanov *et al.*, 2017).

Table 2: Parametric signal characteristics obtained using the 2P and 3P Weibull distributions

Data Point Locations	Weibull (2P)	Weibull (3P)
1	λ=6.9287, c =20.625	λ=3.3645, c=10.958, γ=9.4938
2	λ=4.1397, c=22.735	λ=7.9152, c=2.5730, γ=2.5730
3	λ=8.486, c=25.541	λ=9.8937, c=27.423, γ=1.9385
4	λ=8.6609, c=25.713	λ=11.707, c=30.601, γ=4.9963
5	λ=2.7287, c=21.242	λ=0.81528, c=10.407, γ=9.2

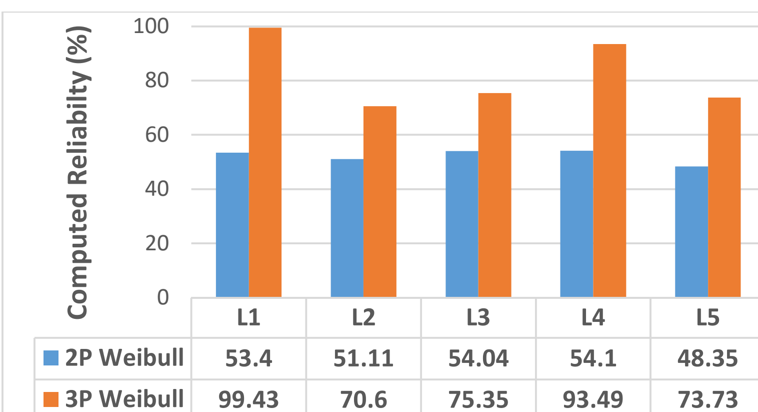

Fig. 2: % Reliability Estimates obtained at different locations using the 2P and 3P Weibull distributions

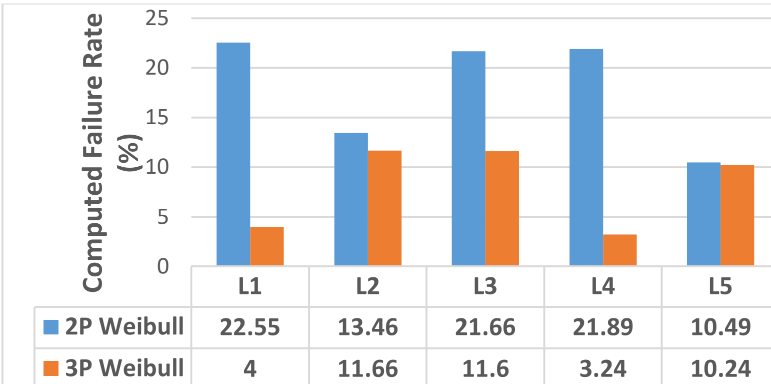

Fig. 3: % Failure Rate Estimates obtained at different locations using the 2P and 3P Weibull distributions

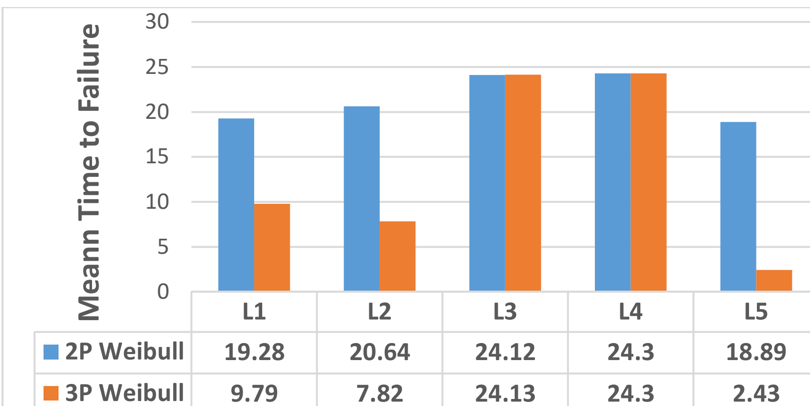

Fig. 4: Mean time before failure Estimates obtained at different Locations using the 2P and 3P Weibull distributions

Table 3: Parametric Signal Characteristics obtained in L1 using the 2P and 3P Weibull distributions

Life Characteristics	Two Parameter	Three Parameter
Reliability	0.5340	0.9943
Mean time to failure (MTTF)	19.28	9.789
Failure rate	0.2255	0.0040

Table 4: Parametric Signal Characteristics obtained in L2 using the 2P and 3P Weibull distributions

Life Characteristics	Two Parameter	Three Parameter
Reliability	0.5111	0.7016
Mean time to failure (MTTF)	20.64	7.82
Failure rate	0.1346	0.1166

Table 5: Parametric Signal Characteristics obtained in L3 using the 2P and 3P Weibull distributions

Life Characteristics	Two Parameter	Three Parameter
Reliability	0.5404	0.7535
Mean time to failure (MTTF)	24.12	24.13
Failure rate	0.2166	0.1160

Table 6: Parametric Signal Characteristics obtained in L4 using the 2P and 3P Weibull distributions

Life Characteristics	Two Parameter	Three Parameter
Reliability	0.5410	0.9349
Mean time to failure (MTTF)	24.30	24.30
Failure rate	0.2189	0.0324

Table 7: Parametric Signal Characteristics obtained in L5 using the 2P and 3P Weibull distributions

Life Characteristics	Two Parameter	Three Parameter
Reliability	0.4835	0.7373
Mean time to failure (MTTF)	18.89	2.43
Failure rate	0.1049	0.1024

For the purpose best model selection, tables IX to XIII show the results obtained after subjecting the 2P and 3P Weibull distribution to three goodness-of-fit (GoF) statistical tests, which are Anderson–Darling, Kolmogorov–Smirnov, and Chi-Square tests. The model with the lowest GoF statistics is the most credible one for reliability and failure rate estimates. From the tables 8 to 12, it is clear that the 3P Weibull distribution is the best one since it provided the lowest GoF statistics across the locations of study.

Table 8: GoF Statistics using 2P and 3P Parameters Weibull Model in L1

GoF Type	Weibull (2P) Statistics	Weibull (3P) Statistics
Kolmogorov Smirnov	0.15	0.07
Anderson Darling	9.18	1.35
Ch-Squared	65.94	21.86

Table 9: GoF Statistics using 2P and 3P Parameters Weibull Model in L2

GoF Type	Weibull (2P) Statistics	Weibull (3P) Statistics
Kolmogorov Smirnov	0.247	0.10
Anderson Darling	2.617	1.70
Ch-Squared	32.98	14.53

Table 10: GoF Statistics using 2P and 3P Parameters Weibull Model in L3

GoF Type	Weibull (2P) Statistics	Weibull (3P) Statistics
Kolmogorov Smirnov	0.17	0.15
Anderson Darling	5.61	4.39
Ch-Squared	42.30	55.80

Table 11: GoF Statistics using 2P and 3P Parameters Weibull Model in L4

GoF Type	Weibull (2P) Statistics	Weibull (3P) Statistics
Kolmogorov Smirnov	0.14	3.13
Anderson Darling	3.40	2.28
Ch-Squared	67.14	37.44

Table 12: GoF Statistics using 2P and 3P Parameters Weibull Model in L5

GoF Type	Weibull (2P) Statistics	Weibull (3P) Statistics
Kolmogorov Smirnov	0.12	0.09
Anderson Darling	2.57	2.06
Ch-Squared	44.20	45.08

CONCLUSION

The fundamental theories of reliability engineering are expressed using probability or probabilistic parameters such as random variables, density functions, and distribution functions. There exist a number of probability distribution functions for describing and fitting a statistical distribution to life reliability data

This work was conducted to evaluate whether the three parameter (3P) Weibull have more satisfactory performance than usual the two-parameter 2P Weibull distributions in the modelling and estimation of LTE broadband network system failure and reliability. To accomplish that, life signal data acquired over LTE broadband network interface at five different locations at Adamkolo Campus of Federal University Lokoja. This is followed by subjecting the acquired data to 2P and 3P Weibull distribution analysis in order to comparatively estimate their reliability and failure rates. To select the best model, both 2P and 3P Weibull distribution were subjected to three goodness-of-fit (GoF) statistical tests, which are Anderson–Darling, Kolmogorov–Smirnov, and Chi-Square tests. Based on the three statistical tests, it was found that the 3P Weibull distribution presented the best reliability and failure estimation performance in all the study locations.

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