

WAVELET SELECTION BASED ON WAVELET TRANSFORM FOR OPTIMUM NOISY SIGNAL PROCESSING

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ABSTRACT

Wavelets transforms (WT), are distinctive set of orthonormal basis functions specially formed for filtering and trend prediction of signal data. One key advantage of WT tool is that it possesses the capacity to cater for both temporal and spectral representation of signal components, simultaneously. However, a key challenge in exploring WT for optimal signal processing is in its ability to determine the right threshold value, which facilitate in the estimation noise level from a noising signal or data. If the chosen threshold value is too big, then some essential components of the signal features may be filtered out. On the other hand, if the chosen threshold value is too small then a substantial quantity of noise will still present in the data. In this work, the optimal signal denoising ability of four thresholding value selection measures, namely Rigrsure, Sqtwolog, Minimax and Heursure are examined and reported on measured signal dataset, under soft thresholding and hard thresholding. The performance of each threshold selection measure is also examined and presented in this work, using five different quantitative evaluation metrics. The metrics includes: the Signal to reconstruction ratio, Root square error, Pulse amplitude distortion, Standard deviation, and mean absolute error. The results reveals that soft thresholding category with Rigrsure repetitively outperforms all other selection rules per every performance metric after denoising, with the heuristic SURE (i.e. heursure) coming in second. The optimal practical signal processing methodology employed in this work can be explored by scientists, engineers and data analyst to trend noisy signal, characterize transient events, compress data and perform many other operations, especially in the area of signal preprocessing in medical imaging and telecommunication system networks.

Keywords: Wavelets, Measured noisy signal, Soft threshing, Hard thresholding, Thresholding selection rules.

INTRODUCTION

Denosing of signals strength sullied by various forms of noise is an established problem in signal processing. The corruption of signal by noise is very common during its production,

transmission, acquisition, reception, processing and reproduction, (Isabona and Azi, 2012). In the past years a number of articles (Ahn *et al.*, 2001; Brown and Churchill, 2001; Ahn *et al.*, 2001; Jwo and Cho, 2010, Ehota *et al.*,

2018, Kim, 2006), have concentrated on removing the noise from the signal in order to enhance the overall signal quality. Conventionally, there exist several noise reduction ways and means, many of which are in frequency or spatial domain via filtering (Chan *et al.*, 2001; Kim, 2006; Ahn *et al.*, 2001; Brown and Churchill, 2001). The conventional ones include, but not limited to the following: the Kalman filter (Jwo and Cho, 2010). High-pass filter, low-pass filter (Ahn *et al.*, 2001), the median filter (Ebhotu *et al.*, 2018) and neural networks based adaptive filter (Baykal and Constantinides, 1997). Nevertheless, all these accustomed approaches have some fundamental inherent limitations. For example, spatial Low-pass filters can smoothen away noise but also will also distort signals power (Brown and Churchill, 2001). Also the high-pass filters possess the ability to improve spatial signal, on the other hand, will also strengthen the background noisy. Another popular conventional noise filtering technique is the Fast Fourier Transform (FFT) (Brown and Churchill, 2001) and (Fei and Yungang, 2011). FFT is principally a low pass filtering method which can cater noisy signal with short time behavior but has bad convergence property and poor time resolution of noisy signal on longer time scales, owing to its short time transform window, (Lokhande, 2017).

The inherent limitation of the above conventional denoising techniques led to the introduction of Wavelet Transform (WT), which combines both the time domain and the standard frequency domain based techniques for potential data analysis. One key advantage of WT tool is that it has capacity to cater for

both temporal and spectral representation of signal decomposition components order simultaneously (Verma and Verma, 2012). It also has simplified and reduced computation complexity algorithms. The prime difference between wavelets and Fourier transform is in their localization. While the former are localized in both time and frequency domain, the latter is only localized in frequency domain. Also, the former is mostly advantageous over the latter one when analyzing some special physical situations and data containing large discontinuities and sharp spikes.

Whereas most of the available literature-Verma and Verma, (2012); Guoshen *et al.* (2007); Nanshan and Mingquan, (2008); Jaishankar and Duraiswamy, 2012; Matko *et al.* (2005); Ojuh and Isabona (2018), focused on Audio and simulated signal denoising using wavelet, all which lacks realistic stochastic nature noisy data. But in this work, the concentration is on practical application of optimal wavelet denoising technique using a realistic measured signal data acquired in operation LTE microcellular telecommunication networks.

Discrete Wavelet Transform

Discrete Wavelet transform has numerous applications like noisy signal removal, transient events characterization, data compression, pattern recognition and medical imaging preprocessing. Specifically, wavelets transform are special mathematical tools for processing data into different coefficients, such that each coefficient at a certain resolution matched to its scale (Guoshen *et al.*, 2007; Nanshan and Mingquan, 2008; Jaishankar and Duraiswamy, 2012). The foremost vital

property of the wavelet transform is to enable and provide means of analysing data at different scales or resolutions. Thus, wavelets can be expressed by the wavelet function $\psi(t)$ and scaling function $\phi(t)$ in the time domain. The wave function and the scaling function

$$P(b, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \psi^t \left(\frac{t-b}{a} \right) p(t) dt \quad (1)$$

In equation (1), ψ^t stands for the complex conjugate of ψ , a is called the scale parameter and b , the position parameter.

If we define $\psi_{a,b}(t)$ as:

$$\psi(a, b) = \frac{1}{\sqrt{a}} \psi \left(\frac{t-b}{a} \right) \quad (2)$$

Which simply implies shifting by b and rescaling by a , then the expression in equation (1) can be written as an inner or scalar product of the signal $p(t)$ with the function $\psi_{a,b}(t)$ as:

$$P(b, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \psi^t_{a,b}(t) p(t) dt \quad (3)$$

where $a \in \mathbb{R}^+$, $b \in \mathbb{R}$, with $a \neq 0$, and $\psi(\cdot)$ satisfies the condition for admissibility.

METHODOLOGY

Data Collection Tools and Settings

The Signal power data used in this paper was acquired from an operational commercial LTE system networks. We utilised a TEMS test investigation software-equipped HP laptop and Samsung Galaxy mobile, both housed in a Rover car with synchronize global positioning system (GPS) to obtain Signal power data from operational commercial LTE system networks. The GPS combined together with the TEMS tools were employed to measure the distance between the LTE receivers and the transmitting base station. The LTE system networks transmit and receive signals over the air interface by means of orthogonal frequency division multiplexing (OFDM). A single LTE cell site is located Waterline area of Port

are also called the mother wavelet and father wavelet, respectively.

Consider a wavelet transform of a real signal $p(t)$; expressing $p(t)$ with respect to wavelet basis function $\psi(t)$, gives:

Harcourt City. The lone site was carefully selected to provide line of site and non-line site features with a good mix of different buildings and structures that characterize a typical urban terrain.

Signal Decomposition, Thresholding and Reconstruction

Signal denoising with wavelet transform entails three steps, namely, decomposition, thresholding and reconstruction. To decompose and reconstruct the signal, we explore the MATLAB 2015a software platform. A 3 wavelet decomposition level is selected to denoise signal dataset. There exist a number of wavelet families in MATLAB wavelet toolbox. But, in the present work only Symlets wavelet family is examined for the reason that it allows for seamless and robust reconstruction of a signal. A

more comprehensive explanation of data decomposition and reconstruction

procedure is contained in the Wavelet Toolbox Users Guide, Version 4.14.1.

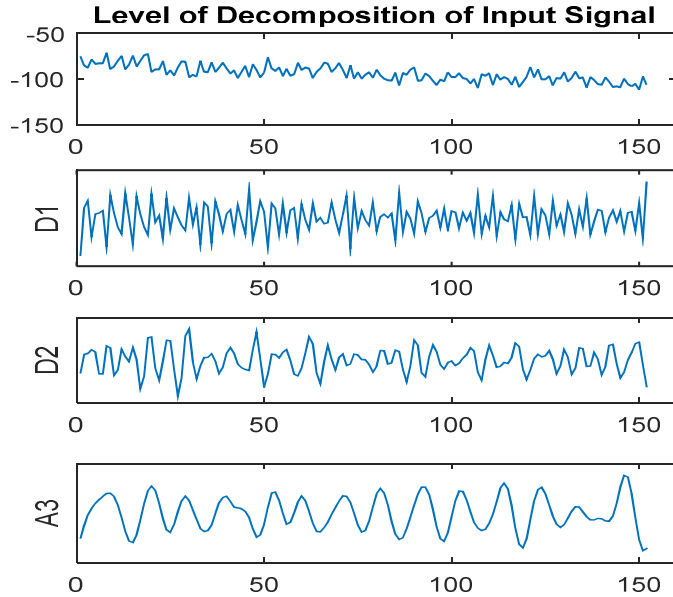


Figure 1: Input Signal Decomposition Level

Thresholding-deals with the extraction of the original signal from the noisy signal form employing various denoising methods. In this paper, the two most common methods signal thresholding, which are a soft thresholding and hard thresholding are considered; their brief description are provided as follows:

- Hard Signal Thresholding = $x(t)$ for $x(t) > x_o(t)$, and 0 for $|x| \leq x_o(t)$
- Soft Signal Thresholding = $\text{sign}(x(t)) (|x(t)| - x_o(t))$ for $x > x_o(t)$, and 0 for $|x_o(t)| < x_o(t)$.

Thresholding Value Selection Rule

A key challenge in thresholding is in determining the best threshold value (λ), which assist in the estimation noise level from a noising signal or data. If the chosen threshold value is too big, then some essential components signal features may be filtered out. On the other hand, if the chosen threshold value is too small then a substantial quantity noise will still present in the data. In this work, four thresholding value selection rules are examined. They Rigrsure, Sqtwolog, Minimax and Heursure. A more comprehensive

description of these four thresholding value selection methods is contained in the Wavelet Toolbox Users Guide.

To compare the impact of each thresholding value selection rule on the measured noisy signal denoising, five different quantitative evaluation metrics are considered for performance evaluation. The metrics includes: the Signal to reconstruction ratio (SRER), Root square error (RMSE), Pulse amplitude distortion (PAD), Standard deviation (STD), and mean absolute error (MAE).

RESULTS AND DISCUSSION

With the aid of MATLAB 2015a software platform, a 3 wavelet decomposition level is selected to denoise the measured signal dataset. All the graphics and computations were also achieved using the MATLAB 2015a software platform. While the plots in figures 2 (a) to 5(a) displayed the measured noisy signals, the various plots in figures 2 (b) to 5 (b) are displayed to reveal the denoising capability of the Rigrsure, Sqtwolog, Minimax and Heursure threshold selection value on measured noisy signal under wavelet-based soft thresholding and hard thresholding. A computed performance summary of the four threshold values election rule with

MAE, RMSE, STD, PAD and SRER in correspondence to soft thresholding and hard thresholding are provided in table 1. Lower values of MAE, RMSE and STD indicate better wavelet denoising performance. On the hand, a higher values of PAD and SRER indicate better wavelet denoising ability. The plotted graphs reveal that "rigrsure" selection rule under soft thresholding constantly outperforms all other rules with every performance metric, with the heuristic SURE (i.e. heursure) coming in second. Table 1 displays the descriptive results summary for all the four threshold selection rules through soft and hard thresholding.

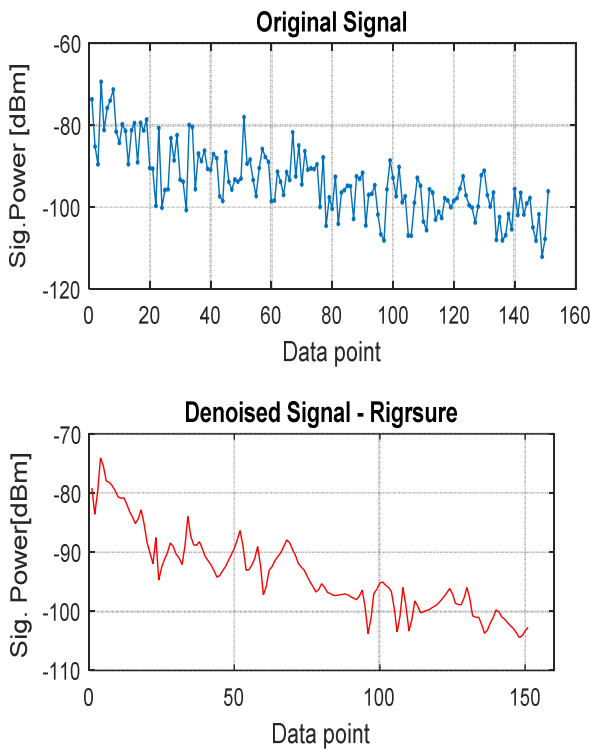


Figure 2 (a): Original and denoised signal with Rigrsure under Soft Thresholding

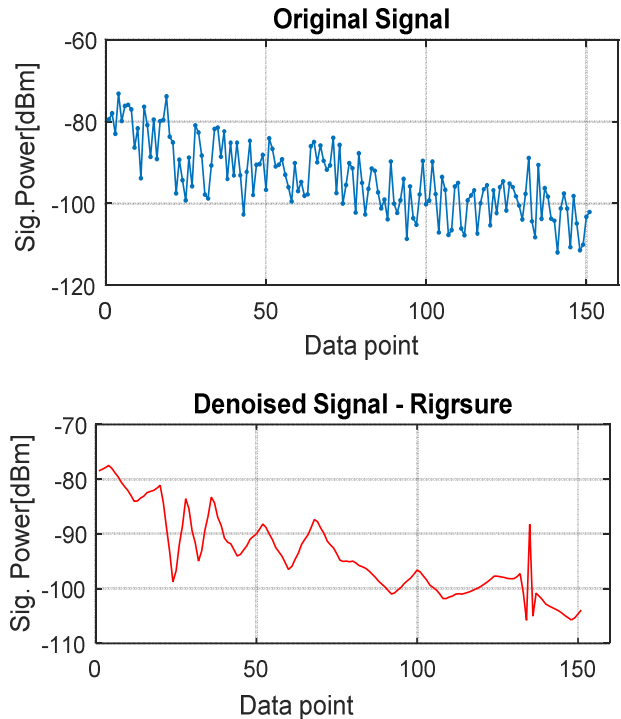


Figure 2(b): Original and denoised signal with Rigrsure under hard Thresholding

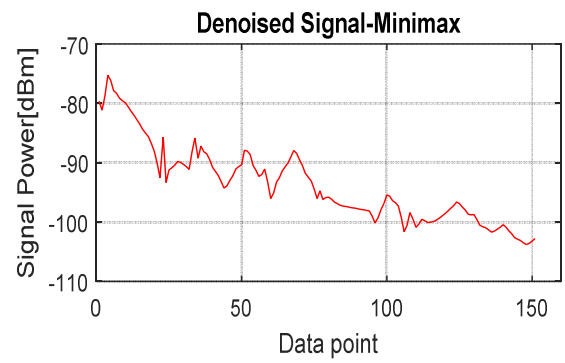
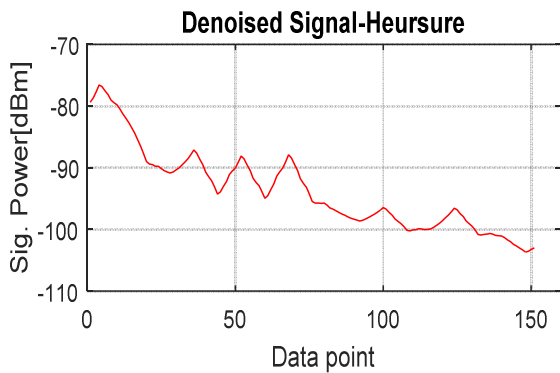
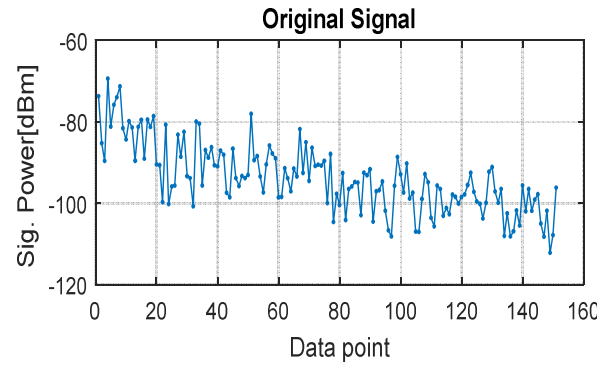
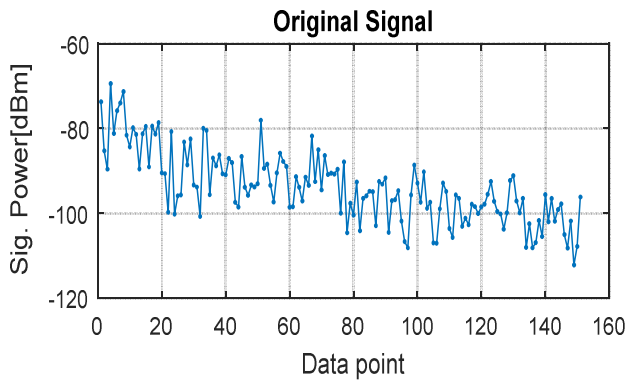


Figure 3(a): Original and denoised signal with Heursure under Soft Thresholding

Figure 4(a): Original and denoised signal with Minimax under soft Thresholding

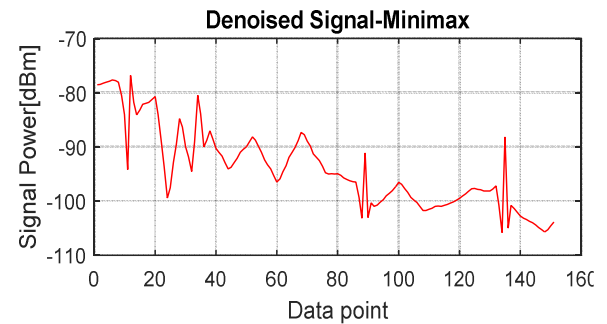
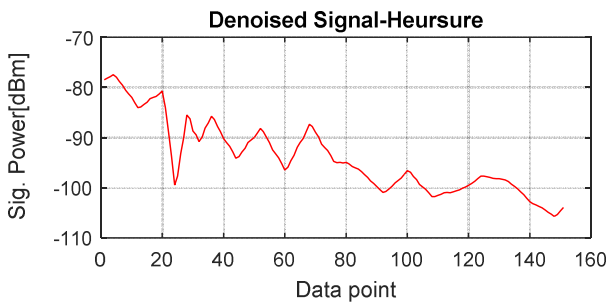
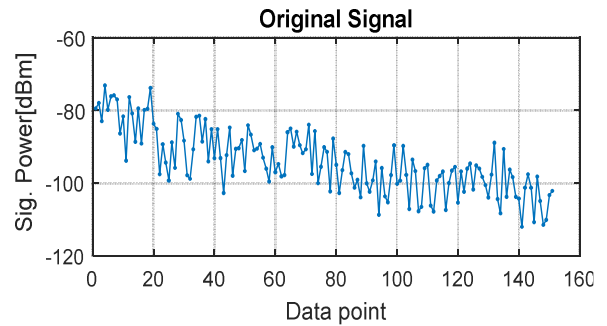
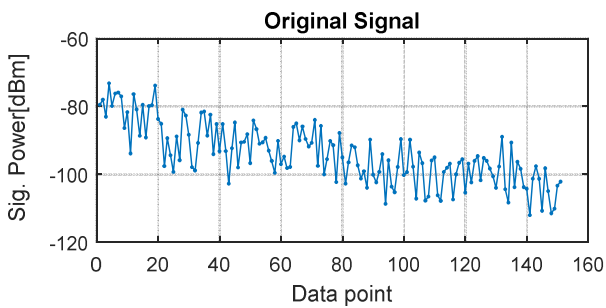


Figure 3(b): Original and denoised signal with Heursure under soft Thresholding

Figure 4(b): Original and denoised signal with Minimax under hard Thresholding

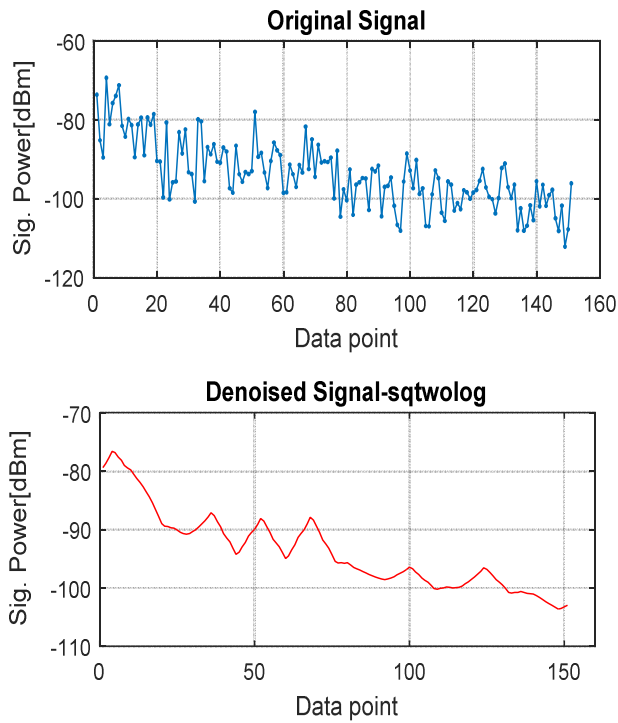


Figure 5(a): Original and denoised signal with sqtwolog under soft Thresholding

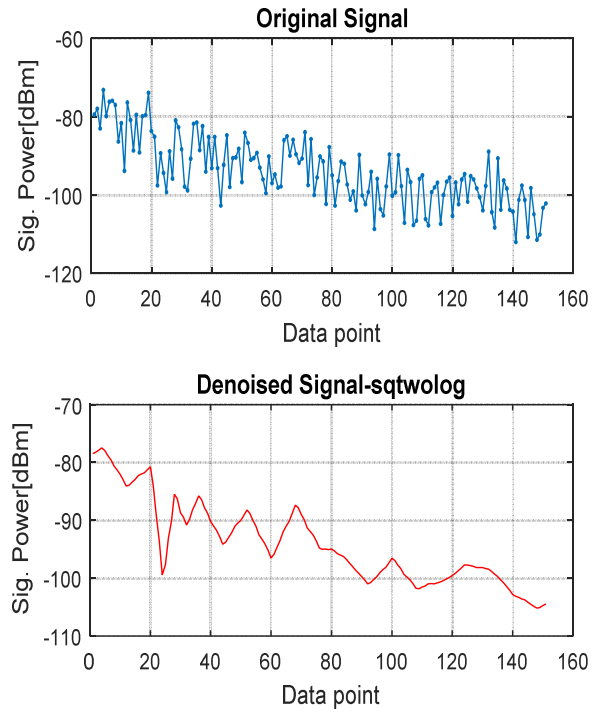


Figure 5(b): Original and denoised signal with sqtwolog under hard Thresholding

Table 1: Result using different Performance Metrics for the four thresholds selection rules under soft and Hard thresholding after denoising

Metric	Threshold selection rule	Soft Thresholding	Hard Thresholding
MAE	Rigrsure	3.52	4.66
	Minimax	4.26	4.10
	Heursure	3.88	3.69
	Sqtwolog	4.25	4.11
RMSE	Rigrsure	4.24	4.66
	Minimax	5.10	4.84
	Heursure	4.59	3.69
	Sqtwolog	5.10	4.86
STD	Rigrsure	2.35	2.83
	Minimax	2.82	3.01
	Heursure	2.45	2.71
	Sqtwolog	2.83	3.02
PAD	Rigrsure	9.91	9.42
	Minimax	11.04	9.54
	Heursure	9.82	9.08
	Sqtwolog	11.05	9.54
SRER	Rigrsure	48.71	47.90
	Minimax	47.11	47.58
	Heursure	48.01	48.40
	Sqtwolog	47.10	47.56

Conclusion

Wavelets are special mathematical tools for processing data into different coefficients, such that each coefficient at a certain resolution matched to its scale. That is, the foremost vital property of the wavelet is to enable and provide means of analysing data at different scales or resolutions. A key challenge in exploring wavelet for optimal signal processing lies one's ability to determine the right threshold value, which facilitate in the estimation noise level from a noising signal or data. If the chosen threshold value is too big, then some essential components signal features may be filtered out. On the other hand, if the chosen threshold value is too small then a

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second. The descriptive results summary for all the four threshold selection rules employing soft and hard thresholding is also presented.

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