BIU Journal of Basic and Applied Sciences 1(1): 1 – 15, 2015.

PROBABILISTIC FIXED LIFETIME INVENTORY MODEL FOR CONTINUOUS DEMAND RATE

***ENAGBONMA, O. AND OBAHIAGBON, K.O.**
Department of Mathematics and Computer Science, Benson Idahosa University, Benin City, Nigeria
* Corresponding author: oenagbonma@biu.edu.ng

ABSTRACT

This paper deals with the development of expected cost model in a fixed lifetime inventory system. In this type of inventory, items in stock are expected to be used before the expiring date. The system consists of an age-wise profile of items (state space). The size of the space is directly proportional to the life of the items. Consequently, the inventory management is faced with lots of challenges otherwise item will be outdated, additional cost such as holding cost, shortage cost and ordering cost will be incurred. Optimal solutions to the fixed lifetime of perishable inventory problem cannot be realized in practice due to their computational complexity arising from the fact that exact formulation of the problem requires information on the age distribution of the items in inventory and the corresponding quantity of items of each age. Hence there is a gap between theoretical results and practical requirements for computational results. The purpose of this study is to minimize the proposed expected cost model, an interval of the optimal value of inventory level (s) was derived Hence we bridge the gap between theoretical results and practical requirements for computational results. The expected holding cost, expected shortage cost, expected outdates cost, ordering cost were computed and were applied to determine the expected cost for the fixed lifetime inventory system.

KEYWORDS: Probability of running out of stock, Expected holding cost, Expected outdate cost, Expected shortage cost, Optimum value of s, Uniform distribution

INTRODUCTION

In this paper, we set up a cost function given as E{C(S)}, for the fixed lifetime inventory system with continuous demand rate. In inventory, items with fixed lifetime in stock are expected to be used before the expiring date, otherwise; such items will be outdated. Examples of such items in real life include blood, drugs, photographic films, chemicals etc. Agbadudu and Enagbonma (2009) compared two expected cost functions in the management of fixed lifetime inventory model. Their result indicates that the proposed model is better than the single period inventory model; since expected costs obtained in the proposed model are less than the expected cost of the single period inventory model. The models were developed if demand has a discrete probability mass function (pmf) and under the deterministic fixed lifetime inventory system. An extension of the study was suggested for fixed lifetime models when demand has a continuous probability density function,.

Chui (1995) developed an approximate expression for the total expected average cost per unit time but could not prove that the cost function was convex. He however gave an iterative scheme for solving the problem. Optimal solutions to the fixed lifetime perishable inventory problem cannot be realized in practice due to their computational complexity arising from the fact that exact formulation of the problem requires information on the age distribution of the items in inventory and the corresponding quantity of items of each age. Hence there is a gap between theoretical results and practical requirements for computational results.

Perry and Posner (1998) proposed an (S-1, S) type perishable inventory system in which the maximum shelf life of each item is fixed, order is placed at each demand time as well as at each time that the maximum shelf life of an item is reached, with the assumption that order lead-times are constant and the demand process for items is Poisson. The method requires the solution of an integral equation with an unknown function. To specify the unknown function,

Williams and Patuwo (1999) also examined the positive lead-time fixed lifetime inventory problem having two periods. The optimal ordering quantity which minimizes the total expected cost function was given as a root of an equation involving multiple integrals. Nahmias (1982) reviewed literatures on problem of determining suitable ordering perishable inventory and inventory subject to continuous exponential decay, both deterministic and stochastic demand for single and multiple products were considered, and both optimal and suboptimal order policies were discussed.

Many authors have considered the fixed lifetime perishable inventory problem with emphasis on blood management. Jagannathan and Sen (1991) reviewed the fixed lifetime perishable inventory literature, a model for determing outdates and shortages crossmatched blood was develop Prastacos (1984) also considered the fixed lifetime perishable inventory literature with emphasis on health-care blood bank inventory perishable items with probability-stochastic model applications. Omosigho (2002) examined the fixed lifetime perishable inventory system and obtained an estimator for the probability that an item will be sold in a given period; the probability is used to derive outdate and shortage quantities. Prastacos (1984) also considered the fixed lifetime perishable inventory literature with emphasis on health -care blood bank inventory perishable items with probability-stochastic model applications. Nahmias (1976) developed a good approximate critical number policy which is easy to compute and easy to use as well. Computational examples entail the approximation to both the best critical number policy and the optimal policy.

Cohen (1976) considered the problem of finding the optimal solution from the class of single critical number order policies for an m-period lifetime perishable product. The steady-state characteristics of the inventory process induced by the order restriction are analyzed. A demonstration of the existence of an invariant measure for an inventory related process was given, this gave information sufficient for cost minimization. Nahmias (1982) reviewed literatures on problem of determining suitable ordering perishable inventory and inventory subject to continuous exponential decay, both deterministic and stochastic demand for single and multiple products were considered, and both optimal and suboptimal order policies were discussed. Taha (2007) considered the single- period models under the assumption that stock replenishment occurs instantaneously. The optimal inventory level is derived based on the minimization of expected inventory cost.

Direct computation of optimal policies was accomplished, some further insight into the form of such policies was provided. Grave (1982) developed two distinct models for studying inventory systems with continuous production and perishable items. Analytical expressions derived from queuing theory are found for the steady-state distribution of system inventory by considering each of the models. Both models assume that the inventory is replenished by a continuous production process. Interaction effects of product lifetime and shortage cost on the proposed fixed lifetime inventory model was considered in Enagbonma and Eraikhuemen, (2010). The results obtained agree with intuition. Enagbonma and Eraikhuemen (2011) also considered the problem of computing optimal ordering policies for a single product with fixed lifetime of exactly m periods. Hwang and Hahn (2000) investigated an optimal procurement policy for items with an inventory level-dependent demand rate and fixed lifetime. A mathematical model and solution methodology were developed. Nandakumar and Morton (1993) developed heuristics from near myopic bounds, which involves viewing periodic inventory problem in the framework of the classic ‘newsboy’ model, various properties of the problem under consideration to derive tight bounds on the newsboy parameters was exploited, computational results were given.

MATERIALS AND METHODS

The proposed model is developed under the following assumptions. The issuing policy is first in, first out (FIFO). Periodic review and order-up-to order policy with parameter S is used. Units expire after the age m periods in the inventory system. Time is divided into discrete periods. The length of a period is arbitrary but fixed. The lead time is zero. Demands in successive periods are independent and identically distributed random variables with known distribution. The sequence of events within each period is as follows: (i) an order is placed and order arrives immediately,(ii) demand for the period is filled and (iii) any unit that has reached the age m and has not been used is outdated.

The order quantity is determined as follows. If IP is the inventory position at the time of placing order, the order quantity is Q = S – IP
The following Mathematical notations were used in the development of the proposed model.

K = setup cost
C = replenishment cost
θ = mean demand
S = Inventory level at the start of each period
h = holding cost
p = shortage cost
i = amount demanded
f(t) = probability density function of the uniform distribution
m = fixed life time
D(t)= demand in period t
E(.) = the expectation function
E (D(t)) = θ
E{C(S)} = Expected total relevant cost function for the period with K Setup cost.
IP = Inventory Position
W = Expected outdate quantity
Q = Expected ordered quantity

The model is based on the probability that an item in the inventory system is sold in a period. The estimator for the probability that an item will be sold is given in Omosigho (2002) as

$$p = \frac{\theta}{s} \tag{1}$$

Let Q be the expectation of units ordered at the end of period t and received at the beginning of period t + 1, given by

$$Q = E\left(Q(t)\right) = \frac{\theta}{\left(1-\left(1-\frac{\theta}{S}\right)^{m}\right)} \tag{2}$$

and W the expectation of units outdated at the end of period t given by

$$W = E\left(W(t)\right) = Q\left(1-\frac{\theta}{S}\right)^{m} \tag{3}$$

Succinctly 1 – P is the probability that an item is not sold in one period. $(1 – P)^{m}$ is the probability that an item will be outdated, this is based on the fact that transaction in the

period S are independent.
In particular outdate decreases with increasing m, since $1 - P < 1$. This result was conjecture by Nahmias (1977). Using (2) in (3) we obtain

$$w = \frac{\theta(1-P)^m}{1-(1-P)^m} \qquad (4)$$

This is an expression similar to that given by Jagannathan and Sen (1991)
for the daily outdates quantity of cross-match blood, P is the proportion of cross-match blood that is actually transfused.

Where D(t) is demand in period t,

$$E\left(D(t)\right) = \theta \qquad (5)$$

$$\text{Ordering cost} = c(S - IP) \qquad (6)$$

$$\text{Expected holding cost} = h\int_0^S (S-t)\ f(t)\ dt \qquad (7)$$

The probability of running out of stock is given by

$$P_{out} = \int_S^\infty f(t)\ dt = 1 - \int_0^S f(t)\ dt \qquad (8)$$

and the shortage quantity is given by

$$Z = \int_S^\infty (t-S)\ f(t)\ dt \qquad (9)$$

$$\text{Expected shortage cost} = p\int_S^\infty (t-S)\ f(t)\ dt = p[\theta - \int_0^S t\ f(t)\ dt - S\,P_{out}] \qquad (10)$$

Applying (8) in (10) yields

$$\text{Expected shortage cost} = p\int_S^\infty (t-S)\ f(t)\ dt$$

$$= p[\theta - \int_0^S t\ f(t)\ dt - S(1 - \int_0^S f(t)\ dt)] \qquad (11)$$

$$\text{Expected outdate cost} = w\int_0^S (S-t)\ f(t) dt \qquad (12)$$

applying (6), (.7), (10) and (12) yields the model given by
The Model

$$E\{C(S)\} = K + c(S - IP) + h\int_0^S (S-t)\, f(t)\, dt + p\int_S^\infty (t-S)\, f(t)\, dt$$
$$+ \; w\int_0^S (S-t)\, f(t)dt \qquad (13)$$

The condition for optimality given by Taha (2007) as

$$E\{C(S-1)\} \geq E\{C(S)\} \qquad (14)$$

$$E\{C(S-1)\} = K + c(S - IP - 1) + h\int_0^{S-1} (S-1-t)\, f(t)\, dt + p\int_S^\infty (t-S+1)\, f(t)\, dt$$
$$+ \; w\int_0^{S-1} (S-1-t)\, f(t)dt \qquad (15)$$

Equation (15) simplifies as follow to (16)

$$= K + c(S - IP) + h\int_0^{S-1} (S-t)\, f(t)\, dt + p\int_S^\infty (t-S)\, f(t)\, dt +$$
$$w\int_0^{S-1} (S-t)\, f(t)dt - h\int_0^{S-1} f(t)\, dt + p\int_S^\infty f(t)\, dt - c - w\int_0^{S-1} f(t)dt$$

$$E\{C(S-1)\} = E\{C(S)\} - h\int_0^{S-1} f(t)\, dt + p\left(1 - \int_0^{S-1} f(t)\right) dt - c - w\int_0^{S-1} f(t)dt$$
$$E\{C(S-1)\} = E\{C(S)\} - h\int_0^{S-1} f(t)\, dt + p - p\int_0^{S-1} f(t))\, dt - c - w\int_0^{S-1} f(t)dt$$
$$E\{C(S-1)\} = E\{C(S)\} + p - c - h\int_0^{S-1} f(t)\, dt - p\int_0^{S-1} f(t))\, dt - w\int_0^{S-1} f(t)dt$$
$$E\{C(S-1)\} = E\{C(S)\} + p - c - (h+p+w)\int_0^{S-1} f(t)\, dt$$
$$E\{C(S-1)\} = E\{C(S)\} = p - c - (h+p+w)P\{t \leq S-1\} \qquad (16)$$

By the condition for a minimum we have

$$E\{C(S-1)\} - E\{C(S)\} = p - c - (h+p+w)P\{t \leq S-1\} \geq 0 \qquad (17)$$

$$p - c \geq (h+p+w)\, P\{t \leq S-1\} \qquad (18)$$

$$(h+p+w)\, P\{t \leq S-1\} \leq p - c$$
$$P\{t \leq S-1\} \leq \frac{p-c}{h+p+w} \qquad (19)$$

The optimum value of S^* must satisfy

$$P\{t \le S^* - 1\} \le \frac{p}{h + p + w} \le P\{t \le S^*\} \tag{20}$$

RESULTS AND DISCUSSION

The ordered and outdate quantities are computed using (2) and (4) for (say) m = 4, 5 and 6, and demand rate $\theta = 35$, S = 37, 38, 39,…,46 the results are given in tables 1, 2, 3, these quantities are applied in the expected cost function

Table 1: Expected Ordered and Outdate Quantities when $m = 4, \theta = 35$

Inventory Level (S)	Expected ordered quantity Q = E(Q(t))	Expected outdate quantity W = E(W(t))
37	35.0003	0.0003
38	35.0014	0.0014
39	35.0039	0.0039
40	35.0086	0.0086
41	35.0161	0.0164
42	35.0270	0.0270
43	35.0420	0.0420
44	35.0614	0.0612
45	35.0856	0.0851
46	35.1148	0.1141

Table 2: Expected Ordered and Outdate Quantities when $m = 5, \theta = 35$

Inventory Level (S)	Expected ordered quantity Q = E(Q(t))	Expected outdate quantity W = E(W(t))
37	35.0000	0.00000162
38	35.0001	0.00000107
39	35.0004	0.00000397
40	35.0011	0.00106800
41	35.0023	0.00023490
42	35.0045	0.00450200
43	35.0078	0.00780300
44	35.0125	0.00125400
45	35.0190	0.00189800
46	35.0274	0.00273900

Table 3: Expected Ordered and Outdate Quantities when $m = 6, \theta = 35$

Inventory Level (S)	Expected ordered quantity Q = E(Q(t))	Expected outdate quantity W = E(W(t))
37	35.0000	0.0000000087
38	35.0000	0.0000085000
39	35.0000	0.0000041000
40	35.0001	0.0000013000
41	35.0001	0.0000034000
42	35.0007	0.0000075870
43	35.0015	0.0015000000
44	35.0126	0.0026000000
45	35.0042	0.0042000000
46	35.0065	0.0065000000

Applying the probability density function (p d f) of continuous demand uniform distribution within the interval [0, S], given in (21) to the model given by (13) we obtain the results given in Tables IV and V using the indicated inventory levels for S = 37, 38,39,40,41,…, 51, and inventory parameters $m = 4$, $\theta = 35$, $c = 120$, $p = 150$, $h = 25$ and values of outdates and ordered quantities given in table 1

$$f(t) = \begin{cases} \frac{1}{S}, & 0 \leq t \leq S \\ 0 & \text{otherwise} \end{cases} \quad (21)$$

To generate results numerically, we have

$$E\{C(S)\} = K + c(S - IP) + h\int_0^S (S-t)\, f(t)\, dt + p\int_S^\infty (t-S)\, f(t)\, dt + w\int_0^S (S-t)\, f(t) dt \quad (22)$$

$$E\{C(S)\} = K + c(s - IP) + h\int_0^S (S-t)\frac{1}{s}\, dt + p\int_S^\infty (t-S)\frac{1}{S}\, dt + w\int_0^s (S-t)\frac{1}{S}\, dt$$

$$E\{C(S)\} = K + c(S - IP) + \frac{h}{S}\int_0^S (S-t)\, dt + p\left[\theta - \frac{1}{S}\int_0^S t\, f(t)\, dt - S\left(1 - \frac{1}{S}\, dt\right)\right] + \frac{w}{S}\int_0^s (S-t) dt$$

set up cost $K = 50$

$$\text{ordering cost} = c(S - IP) = (120)(35.0003) = 4200.036$$

$$\text{Expected holding cost} = \frac{25}{37}\int_0^{37}(37-t)\,dt = \frac{25}{37}\left[37^2 - \frac{37^2}{2}\right]$$

$$= 0.67568(1369 - 684.5) = 462.5$$

$$\text{Expected shortage cost} = 150\left[35 - \tfrac{1}{37}\int_0^{37} t\,dt - 37\left(1 - \tfrac{1}{37}\int_0^{37} dt\right)\right]$$

$$= 150\left[35 - \tfrac{1}{37}\cdot\tfrac{37^2}{2} - 37\left(1 - \tfrac{1}{37}\cdot 37\right)\right] = 150(35 - 18.5) = 2475$$

$$\text{Expected outdate cost} = w\int_0^{s}(s-t)\,\tfrac{1}{s}\,dt$$

$$= \frac{0.0003}{37}\int_0^{37}(37-t)\,dt = \frac{0.0003}{37}\left[37^2 - \frac{37^2}{2}\right] = 0.00553$$

Applying these expected costs into the proposed model we have

$$E\{C(37)\} = 50 + (120)(35.0003) + (462.5) + (2475) + (0.00553)$$

$$E\{C(37)\} = 50 + 4200.036 + 462.5 + 2475 + 0.00553$$
$$E\{C(37)\} = 7187.5415$$

$$\text{set up cost } K = 50$$

$$\text{ordering cost} = c(s - IP) = (120)(35.0014) = 4200.168$$

$$\text{Expected holding cost} = \frac{25}{38}\int_0^{38}(38-t)\,dt = \frac{25}{38}\left[38^2 - \frac{38^2}{2}\right] = 475$$

$$\text{Expected shortage cost} = 150\left[35 - \frac{1}{38}\cdot\frac{38^2}{2} - 38\left(1 - \frac{1}{38}\cdot 38\right)\right]$$
$$= 150(35 - 19) = 2400$$

$$\text{Expected outdate cost} = \frac{0.0014}{38}\left[38^2 - \frac{38^2}{2}\right] = 0.0266$$

Applying these expected costs into the proposed model we have

$$E\{C(38)\} = 50 + (120)(35.0014) + (475) + (2400) + (0.0266)$$
$$E\{C(38)\} = 50 + 4200.163 + 475 + 2400 + 0.0266$$
$$E\{C(38)\} = 7125.189$$

Following the procedure, other values of the expected cost for the fixed lifetime inventory system are obtained for (say) S = 39, 40, 41…,51. The results are given in Table 4

Table 4: Operating Characteristics

Inventory Level. (S)	Setup cost K	Ordering Cost	Expected Holding Cost	Expected Shortage Cost	Expected Outdate Cost	$E\{C(S)\}$ Total Cost
37	50	4200.036	462.5	2475	0.00553	7187.5415
38	50	4200.163	475.0	2400	0.02583	7125.1890
39	50	4200.465	487.5	2325	0.07553	7063.0405
40	50	4201.0256	500.0	2250	0.17094	7001.1966
41	50	4201.9272	512.5	2175	0.32922	6939.7564
42	50	4203.2432	525.0	2100	0.56757	6878.8108
43	50	4205.038	537.5	2025	0.90264	6818.4406
44	50	4207.3649	550.0	1950	1.35020	6758.7152
45	50	4210.2674	562.5	1875	1.92510	6699.6925
46	50	4213.7788	575.0	1800	2.64090	6641.4197
47	50	4217.9234	587.5	1725	3.51010	6583.9340
48	50	4222.7196	600.0	1650	4.54390	6527.2635
49	50	4228.1761	612.5	1575	5.75260	6471.4287
50	50	4234.2978	625.0	1500	7.14540	6416.4432
51	50	4241.0843	637.5	1425	8.73040	6362.3147

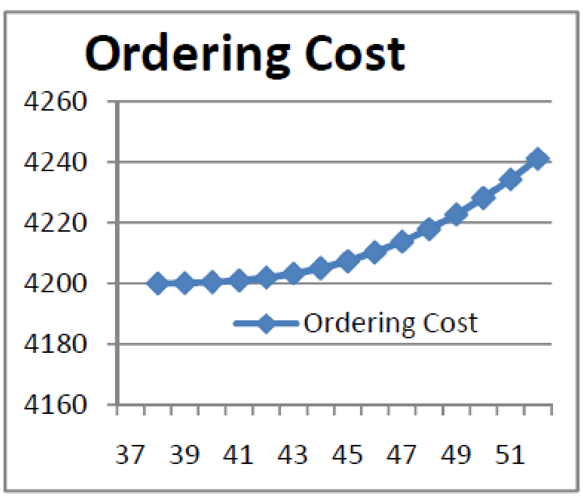

Fig. 1: Ordering cost against inventory level

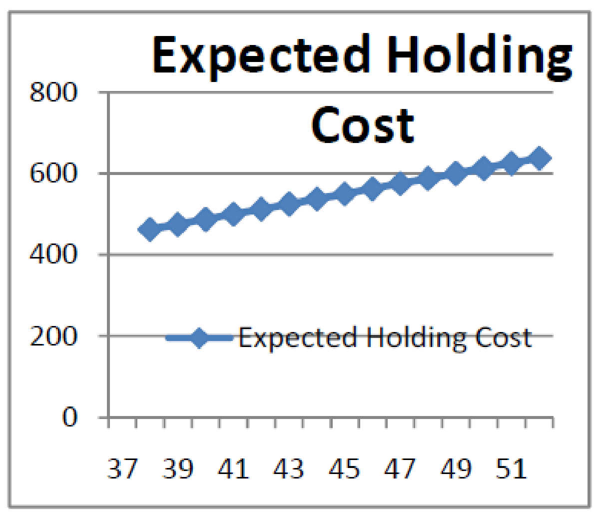

Fig. 2: Expected holding cost against inventory level

Fig. 3: Expected outdate cost against inventory level

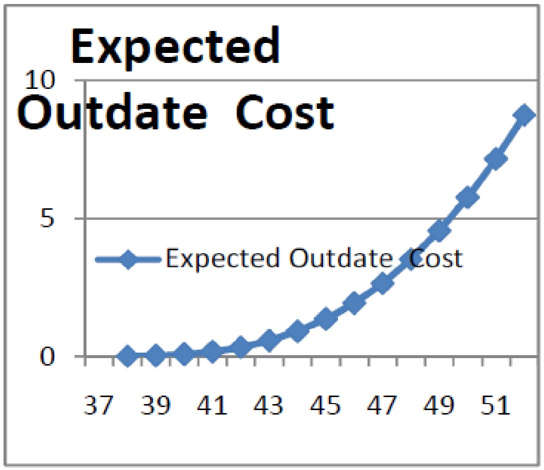

Fig. 4: Expected shortage cost against inventory level

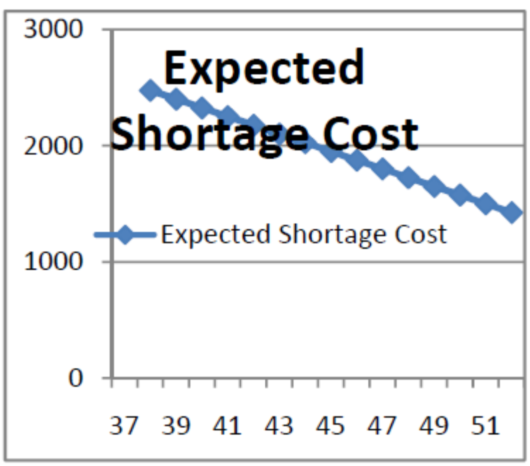

Fig. 5: Setup cost against inventory level

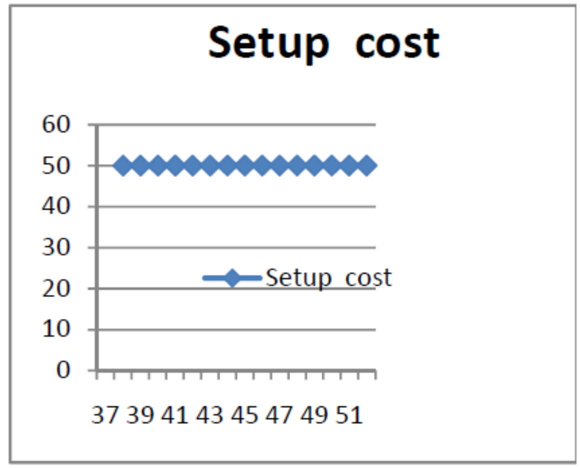

Fig. 6: Total cost function

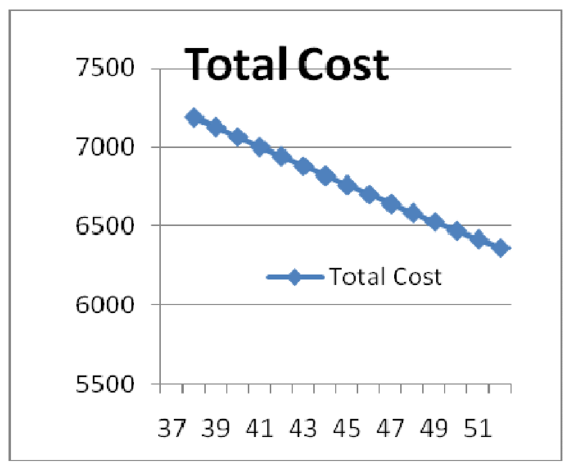

Fig. 7: Expected relevant cost against Inventory Level

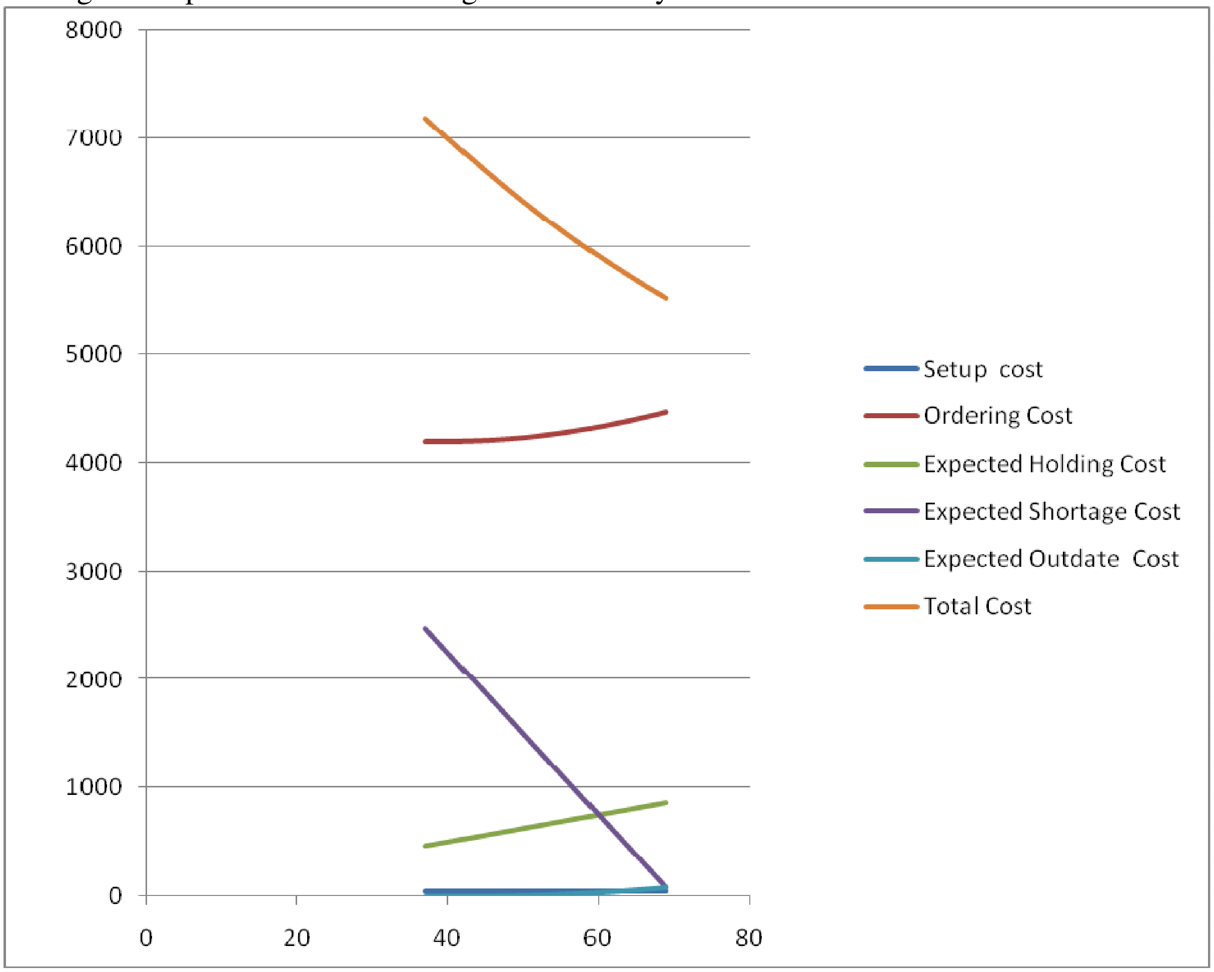

Table 5: Interaction Effects between Inventory Parameters

m \ p	150	180	210	240	270	300
4	(37) 7187.5414	(37) 7682.5414	(37) 8177.5414	(37) 8672.5414	(37) 9167.5414	(37) 9662.5414
5	(38) 7125.0149	(38) 7605.0419	(38) 8085.0149	(38) 8565.0149	(38) 9045.0149	(38) 9525.0149
6	(39) 7062.5057	(39) 7527.5057	(39) 7992.5057	(39) 8457.5057	(39) 8922.5057	(39) 9387.5057
7	(40) 7000.0023	(40) 7450.0023	(40) 7900.0023	(40) 8350.0023	(40) 8800.0023	(40) 9250.0023
8	(41) 6937.501	(41) 7372.501	(41) 7807.501	(41) 8242.501	(41) 8677.501	(41) 9112.501
9	(42) 6875.0005	(42) 7295.0005	(42) 7715.0005	(42) 8135.0005	(42) 8555.0005	(42) 8975.0005
10	(43) 6812.5003	(43) 7217.5003	(43) 7622.5003	(43) 8027.5003	(43) 8432.5003	(43) 8837.5003
11	(44) 6750.0001	(44) 7140.0001	(44) 7530.0001	(44) 7920.0001	(44) 8310.0001	(44) 8700.0001
12	(45) 6687.5001	(45) 7062.5001	(45) 7437.5001	(45) 7812.5001	(45) 8187.5001	(45) 8562.5001

In the results presented in Table 4, we considered the product lifetime m = 4, shortage cost p = 150, fixed parameters are θ = 35, c =120, h = 25, K = 50. Inventory level S = 37, 38, 39, ….51 Applying these data into the proposed expected cost we have the results depicted in Table 4. The entries in Table 4 indicates a positive directional relationship between (a) ordering cost and inventory level, (b) expected holding cost and inventory level, (c) expected outdate cost and inventory level. However an inverse relationship exists between expected shortage cost and inventory level.

Graphically, fig. 1 - 3 indicate a positive directional relationship between (a) ordering cost and inventory level, (b) expected holding cost and inventory level ,(c) expected outdate cost and inventory level . However fig. 4 indicates an inverse relationship between expected shortage cost and inventory level, figures 5 and 6 indicates setup cost and total cost respectively. Fig. 7 depicts expected relevant costs against inventory level.

The results presented in Table 5 agree with what one would expect. We were particularly interested in the interaction effects of the product lifetime and shortage cost on the inventory system with fixed lifetime. So we considered nine values of the product lifetime (m = 4, 5, 6,…,12) and six values of shortage cost (p = 150, 180, 210, 240, 270, 300). Fixed parameters are θ = 35, c = 120, h = 25. For each set of parameters we reported the inventory level (s), followed by the expected cost of using the inventory level .The results in Table 5 revealed that the expected cost increases with shortage p. Also as the lifetime of m increases, expected cost decreases.

In conclusion, we have been able to bridge the gap between theoretical results and practical requirements for computational results. We computed the ordering cost, expected holding cost, expected shortage cost, expected outdates cost, and these computations were applied to determine the expected cost for the fixed lifetime inventory system. Finally, organization may like to operate under a given aspiration scenario, values of inventory level that satisfy such condition are identified and are used in the cost function to determine optimal operating conditions, this will go a long way to reduce waste and holding cost.. Researches about the optimal ordering policies for this type of inventory system with fixed lifetime could be suggested for further studies. Models of this sort are currently being investigated and will be reported in the future. The computations were facilitated using a computer program.

REFERENCES

Agbadudu, A. B. and Enagbonma.O. 2009. Comparison of two expected cost functions in the management of fixed lifetime inventory model " *Abacus the Journal of Mathematical Association of Nigeria*, 36(2): 147-156.

Chui, H. N. 1995. An approximation to the continuous review inventory model with perishable items and lead times. *European Journal of Operations Research*, 87: 93 – 108.

Cohen, M.A. 1976. Analysis of single critical number ordering policies for perishable inventories. *Operations Research*, 24(4): 726 -741.

Enagbonma. O. and Eraikhuemen, I. B. 2010. Interaction effects of product lifetime and shortage cost on the proposed fixed lifetime inventory model. *South East Asian Journal of Mathematics and Mathematical Sciences*, 8(3): 11-23.

Enagbonma. O and Eraikhuemen I. B. 2011. Optimal Ordering Policies for the Inventory System With Fixed Lifetime. *Australian Journal of Basic and Applied Sciences*, 5(12): 3343-3348.

Graves, S.C. 1982. The application of queuing theory to continuous perishable inventory systems", *Management Science*, 28(4): 400 - 406.

Hwang, H. and Hahn, K. 2000. An optimal Procurement policy for items within inventory level-dependent demand rate and fixed lifetime. European *Journal of Operations Research*, 87: 93-108.

Jagannathan, R. and Sen, T. 1991. Storing cross matched blood: A perishable inventory model with prior allocation. *Management Science*, 37(3): 251- 266.

Nahmias, S. 1976. Myopic approximations for the perishable inventory problem. *Management Science*, 22(9): 1002 -1008.

Nahmias, S. 1982. Perishable Inventory Theory: A review", Operation Research, 30(4): 680 – 708.

Nandakumar, P. and Morton, T. E. 1993. Near Myopic heuristics for the fixed life perishable problem", *Management Science*, 39(12): 1490 – 1498.

Omosigho, S. E. 2002. Determination of outdate and shortage quantities in the inventory problem with fixed lifetime. Intern. J. Computer Math., 79(11): 1169 – 1177.

Perry, D. and Posner, M. J. M. 1998. An (S-1, S) inventory system with fixed shelf life and constant lead times. Operations Research, 46(Supp. 3): 565 -571.

Prastacos, G. P. 1984. Blood inventory bank management. An overview of theory in practice. *Management Science*, 30(7): 777 – 800.

Taha, H. A. 2007. Operations Research. Prentice – Hall of Indian Private limited, New Delhi.

Williams, C. L. and Patuwo, B.E. (1999)" A perishable inventory model with positive order lead times", *European Journal of Operational Research*, 116: 352 – 373.